PIV investigation of a turbulent boundary layer over a three-dimensional rough wall

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Riassunto. Sono state eseguite misure in uno strato limite turbolento su una parete con rugosità tridimensionale per mezzo di tecniche PIV (Particle Image Velocimetry) a basso numero di Reynolds (Re₀=1234). La rugosità è costituita da elementi piramidali aventi un rapporto tra lo spessore dello strato limite e la loro altezza pari a δ/k = 17.2. Le misure sono state eseguite in un piano normale alla parete e parallelo alla corrente esterna allo strato limite. Misure del campo medio di velocità e di turbolenza confermano la validità dell’ipotesi di similitudine di Townsend nonostante il relativo alto valore dell’altezza della rugosità e il basso numero di Reynolds. L’analisi delle funzioni di autocorrelazione dimostrano che fuori dallo strato rugoso l’organizzazione dei moti turbolenti è qualitativamente e quantitativamente simile a quella di uno strato limite su parete liscia. Al contrario, la corrente nel sottostrato rugoso mostra una perdita di coerenza longitudinale, suggerendo che l’organizzazione della turbolenza vicino alla parete è fortemente influenzata dalla presenza degli elementi rugosi. La visualizzazione del flusso nel sottostrato rugoso mostra la non omogeneità del campo di moto nella direzione della corrente.

Infine, la rappresentazione media del flusso mostra la presenza di moti vorticosi nella regione subito a valle dell’apice della piramide. Si ritiene che queste strutture vorticose si originino dall’interazione della corrente fluida con gli elementi rugosi.

Parole chiave: turbolenza, rugosità di parete, strato limite, PIV.

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Abstract. Particle image velocimetry (PIV) measurements at relatively low Reynolds number ($Re_{\theta}=1234$) in a turbulent boundary layer over a 3D roughened surface, consisting of pyramidal rows, are presented and compared to those for a smooth wall. The value of $\delta/k = 17.2$, characterizing the present roughness geometry, is much lower than the corresponding values in most of published papers. Measurements have been taken in a streamwise wall normal plane. Mean flow and Reynolds stresses measurements support the Townsend’s wall similarity hypothesis in spite of the relatively high value of the roughness thickness and the relatively low Reynolds number. The analysis of two point streamwise velocity autocorrelation functions demonstrates that, out of the roughness layer, the turbulent organized motions behave, qualitatively and quantitatively, in complete similitude for the smooth and the rough walls. Conversely, the flow in the roughness sub-layer shows a loss of longitudinal coherence, suggesting that the near wall turbulence organization is strongly affected by the roughness elements.

The visualization of the flow in the inner roughness sub-layer show the non homogeneity of the flow in the streamwise direction. In particular, the behaviour of the mean and fluctuating quantities around the roughness elements is described.

Finally, the mean representation of the flow identifies swirling motions in the region and just downstream of the pyramid apex. It is argued that they are the signature in the measurement plane of unstable vortical three dimensional structures, arising from the interaction of the incoming flow with the roughness elements.

Keywords: turbulence, wall roughness, boundary layer, PIV.

1. Introduction

Understanding the effect of roughness on wall bounded turbulence is important mainly for the two following reasons: i) the validity of the Townsend’s similarity hypothesis, ii) the understanding how the wall roughness perturbs the near wall turbulence organization. Moreover, other questions related to the two previous points arise: the use of wall functions for rough wall boundary layer numerical flow simulations is appropriate or not? The assumption that the small turbulent scales have universal behaviour (as for instance is assumed in large eddy simulation) is valid for turbulent boundary layers on rough walls? The techniques developed for wall flow control are also suitable for the case of rough walls?
According to the wall similarity hypothesis of Townsend (1976) and the subsequent extensions by Perry and Chong (1982) and Raupach et al. (1991), at sufficiently high Reynolds numbers, outside the roughness sublayer extending about five roughness heights, \( k \), from the wall, the turbulent motions are independent of the surface conditions. This implies that the mean velocity profiles for smooth and rough walls expressed in velocity defect form and the turbulent stress profiles, both normalized with respect to inner variables, collapse outside the roughness sub-layer, when displayed in function of the distance from the wall normalized by the boundary layer thickness, \( y/\delta \). A general agreement on the validity of this outer-layer similarity hypothesis has not yet been conclusively found.

A large amount of papers has been published on the general subject of a turbulent boundary layer over two and three-dimensional rough surfaces, since the pioneering work of Nikuradse (1933) and later on of Perry et al. (1969). The latest review of Jimenez (2004) covers in a comprehensive way most of the work published on the subject until the 2004. More recent papers will be cited in the following. Bakken et al. (2005) showed that very little roughness effects can be observed outside five roughness length scales for different geometries in the mean flow and also in Reynolds stresses and third-order moments, when scaling with friction velocity is used. They tested in a two-dimensional channel flow two surfaces roughened by two-dimensional rods (\( h/k = 29.4 \), \( h \) is the half channel height and \( p/h = 8 \), \( p \) is the rod spacing) and by three-dimensional mesh roughness consisting in perforated plates (\( h/k = 33 \)), for a wide range of Reynolds numbers, going from \( Re_c = 360 \) to \( Re_c = 3300 \). \( Re_c \) is the Reynolds number based on the friction velocity and on the half channel height. They found that the turbulence integral length scale in the streamwise direction was shorter for the case of rough surfaces with respect to the case of a smooth wall, indicating a break-up of the near-wall streamwise vortices over the rough wall. At the same time they observed the turbulence structures moved outwards a distance corresponding approximately to the roughness height and they showed by quadrant analysis that ejection/sweep processes were very similar for smooth and rough walls. Part of the 2D-roughness results shown in Bakken et al. (2005) were also reported in Krogstad et al. (2005), namely the measurements at \( Re_c = 670 \) for the smooth wall and \( Re_c = 600 \) for the 2D-roughened wall. It should be pointed out that the Bakken et al. and Krogstad et al. experiments were performed in a channel flow, where the conditions are somewhat different from the case of a boundary layer. In a channel flow there is no entrainment from the external flow, the pressure gradient along the wall is negative, the outer-layer length scale (\( h \)) and the
friction velocity are constant along the channel. All these and other factors, as discussed in Krogstad et al. (2005), suggest that there is a little possibility for the velocity defect profiles to be different in the rough and smooth cases and that the turbulent production (properly normalized) cannot be very different in the outer-layers of smooth and rough wall channel flows. Schultz and Flack (2005) performed turbulent boundary layer measurements on a flat plate, covered with uniform spheres and also on the same surface with the addition of a fine scale grid roughness ($3000 \leq Re_\theta \leq 14000$). $Re_\theta$ is the Reynolds number based on the momentum thickness and the external velocity. The roughness heights were considerably lower than the boundary layer thickness. The Authors show that the mean velocity profiles for all the surfaces collapse well in the velocity defect form. The Reynolds stresses for the two rough surfaces also show good agreement throughout the entire boundary layer and collapse with smooth wall results outside of the roughness sub-layer (for $y > 5k$). Kunkel and Marusic (2006) carried out measurements in the log region of an atmospheric turbulent high Reynolds number boundary layer. The site ground was characterized by $k_s/\delta = 14.5 \cdot 10^{-6}$ and $k_s/\delta = 79.5 \cdot 10^{-6}$, where $k_s$ is the Nikuradse equivalent sand grain roughness height. Their results show good support for the Reynolds number similarity hypothesis, comparing results with Reynolds numbers varying over three orders of magnitude. Shockling et al. (2006) found that the mean velocity profiles for smooth and rough pipes are similar in the outer-layer for a very large range of Reynolds numbers and for very small values of the surface $k_{rms}$. Connelly et al. (2006) tested a wide range of roughness geometries and roughness heights. The model surface was covered with sandpapers and with woven wire meshes implying values of the ratio $\delta/k$ from 16 to 110 and values of $Re_\theta$ from 6500 to 12500. They found that the mean velocity profiles for all the test surfaces, including the cases of the lowest values of $\delta/k$, agree in the velocity defect form in the overlap and outer-layer, when normalized by the friction velocity. This result makes evident the fact that the roughness effects on the mean flow are confined to the inner sub-layer ($y < 5k$) and outer-layer similarity of the mean velocity profile applies even for relatively large roughness. The more recent paper of Flack et al. (2007) extended the Connelly et al. results to Reynolds stress and third order moment measurements. They observed similarity also in the turbulence quantities between smooth and rough surfaces beyond $5k$ or $3k_s$ from the wall. Considering the relatively high values of the tested roughness heights, the Authors concluded that a critical roughness height, where the roughness affects all the boundary layer, does not exists. The concept is that extending $5k$ (or $3k_s$) from the wall, the
roughness layer begins to occupy an ever increasing fraction of the outer-layer, with the outer flow only gradually modified. The turbulence structure, through spectra of the fluctuating velocity components, swirl strength and two-point auto- and cross-correlations of fluctuating velocity and swirl, was experimentally studied by Volino et al. (2007) comparing results on a woven wire mesh surface and a smooth surface. The ratio $\delta k$ for the roughened wall was 71 and the $Re_\theta$ for the smooth and rough walls were 6069 and 7663 respectively. Normalizing quantities using outer variables, they found that quantitative similarity between the smooth wall and the rough wall flows holds in large part. The results were in good agreement, both qualitatively and quantitatively, with the turbulence structure for smooth wall boundary layers documented in the literature. The boundary layer was characterized by packets of hairpin vortices which induce low speed regions with regular spanwise spacing. The same types of structure are observed for the rough and smooth wall flows. Wu and Christensen (2007) tested a three-dimensional roughened wall replicated from a surface scan of a damaged turbine blade with the average peak-to-valley roughness height $k=\delta 28$ and $k=\delta 50$, at $Re_\theta \approx 13000$. They found the mean velocity deficits along with the Reynolds normal and shear stress profiles, for both roughness conditions, collapse on the smooth wall baseline in the outer-layer when appropriately scaled by the friction velocity, $U_\tau$, fully supporting Townsend’s wall similarity hypothesis. Probability density functions and quadrant analysis of the instantaneous events contributing to the mean Reynolds shear stress show similar outer-layer consistency between the smooth and rough cases when scaled appropriately with $U_\tau$. Moreover, they found one-dimensional, two-point streamwise, and wall-normal velocity autocorrelation coefficients to collapse in the outer region, indicating a similarity in the spatial structure of the outer-layer turbulence. Coherent structures in turbulent channel flows over wavy surfaces were investigated by Kuhn et al. (2007), at a bulk Reynolds number of 11200. They performed proper orthogonal decomposition of the three velocity components to specify the influence of the different surface geometries on the coherent motions. They tested sinusoidal wall profiles with different amplitude-to-wavelength ratios finding similar large scale structures in the vicinity of the surface. These structures were identified as streamwise-oriented, counter-rotating vortices, having comparable scales, not directly linked to the surface wave amplitude, nor to the wavelength. Mean flow profiles, skin friction, and integral parameters for boundary layers developing naturally over a wide variety of fully aerodynamically rough surfaces are presented and discussed by Castro (2007). A very wide range of the ratio of roughness element height to
boundary layer thickness \((0.03 < k/\delta < 0.5)\) is covered. It is shown that irrespective of the nature of the roughness, unless the surface is very rough, all the data collapse, even if the typical roughness element height exceeds some 50% of the boundary layer momentum thickness. Vesely et al. (2009) tested a two-dimensional roughness consisting of grooves perpendicular to the flow showing excellent agreement in the outer-layer between the roughened wall and the smooth wall. Mean flow expressed in velocity defect form and inner scaled streamwise Reynolds stresses were found to overlap within the measurement uncertainty in spite of the relatively high value of the roughness thickness \((\delta/k = 28.6; \delta/k_s \approx 7)\) and the relatively low Reynolds numbers, \(Re_\theta = 930\) and 1360, respectively for the smooth and rough wall. Vesely et al. reported that the organization in the buffer layer of the flow in longitudinal highly coherent high- and low-speed streaks is strongly influenced by the two-dimensional wall roughness and the velocity streaks are weakened or even destroyed. Conversely, in the outer-layer the flow over the roughened surface shows an organization similar to the one for the smooth wall. Two points streamwise velocity autocorrelation functions for the rough wall suggest the occurrence of packets of vortical structures that are coherently aligned with an inclination angle (with respect to the wall) very close to the one found for the smooth wall. Also the streamwise and spanwise scales of the packets and of the single vortices are shown to be very similar in the outer-layer for the rough and the smooth wall flows.

Wind tunnel measurements of turbulent boundary layers over three-dimensional rough surfaces have been carried out to determine the critical roughness height beyond which the roughness affects the turbulence characteristics of the entire boundary layer by Amir and Castro (2011). Experiments were performed on three different roughness configurations, square random height elements, diamond-pattern wire mesh and a sandpaper type grit, with \(Re_\theta\) ranging from 1300 to 28000. A wide range of the ratio of roughness element height \(k\) to boundary layer thickness \(\delta\) was covered (from 0.04 to 0.40). For all the surfaces the mean profiles collapse well in velocity defect form up to large values of \(k\) \((k/\delta \approx 0.2)\). Up to \(k/\delta \approx 0.15\) the Reynolds stresses for all surfaces show good agreement throughout the boundary layer, collapsing with smooth wall results outside the near-wall region independently from \(Re_\theta\). With increasing \(k/\delta\), however, the turbulence above the near-wall region is gradually modified until the entire flow is affected. In the near wall region they found a noticeable increase in stress contributions from strong sweep events even at quite low \(k/\delta\).
In contrast with the previous cited papers, direct numerical simulations of turbulent channel flow with one smooth wall and one covered with regular three-dimensional roughness elements by Bhaganagar et al. (2004) show that roughness effects can be observed in the outer-layer. They compare results from the smooth and rough wall sides of the channel for three different roughness heights of \( k^+ = 5.4, 10.8 \) and 21.6 (where the apex \( ^+ \) indicates quantities scaled with the inner variables), for \( Re_\tau \) of 400. The roughened wall consisted of smooth 3D egg carton-shaped surface. They found that the roughness alters the velocity fluctuations in the outer-layer, while the vorticity fluctuations are relatively unaffected. In addition, the higher-order moments and the energy budgets demonstrate significant differences between the smooth-wall and rough-wall sides. More recently, Djenidi et al. (2008) tested square bars placed transversely to the flow in a turbulent boundary layer, for two different ratios between the bar spacing and the bar height, \( \lambda/k = 8 \) and 16 and for two values of \( \delta/k = 20 \) and \( \delta/k = 38 \) (\( Re_\theta = 5300 \)). The results showed that the roughness density effects were felt well beyond the near wall region of the flow. Djenidi et al. observed unstable quasi-coherent structures in the form of spanwise vortices to take place in the near wall region. These vortical structures were seen to be formed at the trailing edge of the roughness elements through the interaction of the incoming flow and the almost stagnant flow behind the roughness and convected downstream. Further, the Authors argued that the turbulence production mechanism was related to the formation of these vortices and to their interaction with the overlying flow. In Volino et al. (2009) turbulence measurements for a zero pressure gradient boundary layer over a two-dimensional roughness are presented and compared to previous results for a smooth wall (Volino et al., 2007). The two-dimensional roughness consisted of transverse square bars having the height, \( k \), of 0.03\( \delta \) and the bar spacing of 8\( k \). Although the mean flow is not significantly affected by the wall roughness, the two-dimensional bars lead to significant changes in the turbulence in the outer flow. Reynolds stress profiles, particularly wall-normal Reynolds normal stress and Reynolds shear stress, increase. This difference in the Reynolds stresses was attributed to large-scale turbulent motions emanating from the wall. Recently Volino et al. (2011) tested turbulent boundary layers over surfaces roughened by periodic small 2D-bars and large 2D-bars (\( \delta/k = 161, Re_\theta = 4984 \) and \( \delta/k = 31, Re_\theta = 4260 \) respectively) in comparison with a 3D-roughened wall (staggered cubes, \( \delta/k = 28, Re_\theta = 4952 \)) and a smooth wall (\( Re_\theta = 6069 \)). The results show outer-layer similarity between the cases with three-dimensional roughness and smooth walls, and deviations from similarity in cases with small and large...
two-dimensional transverse bars (see also Volino et al., 2009). Differences are most apparent in correlations of turbulence quantities, which are of larger spatial extent for the rough-wall cases. The small two-dimensional bars have a larger effect than the staggered cubes, in spite of the bar height being only 1/7 of the cube height. The observed differences between the rough and smooth wall results are attributed to large-scale attached eddies which extend from the roughness elements to the edge of the boundary layer.

The present work deals with a turbulent boundary layer over a three-dimensional roughened wall and it follows a previously published paper reporting results about a flow over a two-dimensional rough surface (Vesely et al., 2009). PIV data obtained for a three-dimensional geometry of the wall roughness, having a relatively low value of $\delta/k$ ($\delta/k = 17.2$), are here shown and commented in comparison with smooth wall data. The wall roughness, consisting of pyramidal rows, was tested at relatively low Reynolds number, $Re_\theta = 1234$. The choice of moderately large value of the roughness height $k$ and of low Reynolds number was due to the willingness of generating a flow configuration in which the influence of the wall roughness on the turbulent structures may be expected, according to Jimenez (2004), to be noticeable. As in Vesely et al. (2009), an important motivation of the research was to try to give a contribution to the understanding of the behaviour of the turbulent organized motions in a boundary layer over a rough wall when compared with the case of a flow over a smooth surface. Results are shown in the whole boundary layer, including the roughness sub-layer region.

In recent years other investigations have been published having as object the study of the flow over pyramidal rough walls. The geometry of this roughness can be simply described in terms of the pyramid’s height, orientation with respect to the flow direction, face slope and roughness density.

Schultz and Flack (2009) presented results of an experimental investigation of the flow over pyramidal rough walls where both the pyramidal height and the slope were systematically varied. The mean velocity profiles for all the rough surfaces collapse with smooth wall results when presented in velocity defect form, supporting the use of similarity methods. The results for the steepest surfaces indicate that the roughness function, $\Delta U^+$, scales almost entirely on the roughness height with little dependence on the slope of the pyramids. The steepest slope of the pyramid surface was 45°, with $\delta/k$=50 and $Re_\theta$ varying from 3960 to 30100. Hong et al. (2011) examined the flow structure and turbulence in a pyramidal rough wall channel flow with $h/k \approx 50$, for $Re_f=3520\div5360$. They found that the spatial variation in the mean flow, Reynolds stresses, turbulent kinetic
energy (TKE) production and dissipation rates are confined to a distance from the wall \(< 2k\). Instantaneous realizations show that eddies having the same scale of the roughness are generated near the wall and lifted up rapidly by the large-scale structures (hairpin packets) that populate the outer layer. Consequently a small scale roughness signature is present across the entire channel. More recently, Hong et al. (2012) extended the previous work focusing especially on the turbulence structure and its role in sub-grid-scale energy transfer. To explain the observed flow signature they proposed U-shaped quasi-streamwise vortices developing near the top of the pyramids as spanwise vorticity is stretched in regions of high streamwise velocity between the pyramidal roughness elements. The flow induced by adjacent legs of the structures causes powerful ejections, that lift these vortices away from the wall. The existence of these structures has been also supported by Talapatra and Katz (2012) by high resolution holographic PIV measurements on a wall roughened by distributed pyramids at $Re_I=3520, h/k = 54$.

The cited works having as object the study of the turbulent boundary layer over a pyramidal roughened wall deal with the flow in the roughness sub-layer, showing results from the top of the pyramids up to the external part of the roughness sub-layer. In the present contribution the measurements are extended to the more internal region close to the base of the pyramids.

2. Experimental setup

The experiments were carried out in the water tunnel at the Dipartimento di Ingegneria Meccanica e Aerospaziale of the Politecnico di Torino. This facility is a closed-loop open surface channel with a 350 mm wide, 500 mm high, and 1800 mm long test section. The free-stream turbulence level was 1.2% and the maximum ratio of the boundary layer thickness to the width of the tunnel was 0.15, ensuring that the mean boundary layer on the flat plate was 2D over the central 70% of its width. Measurements were taken on a flat plate with a length of 2050 mm (Fig. 1a), in a region about 1750 mm downstream the leading edge for the canonical boundary layer case and about 500 mm downstream the leading edge for the boundary layer developing over the 3D rough wall. The pressure gradient was null along the test section. A three-dimensional roughness, consisting of pyramids with the diagonal of the square base oriented in the direction of the mean flow, was glued over the surface of the flat plate. In Fig. 1b the geometrical characteristics of the wall roughness are reported. The roughness height and geometry were selected to be significative for the case of moderately small
values of $\delta k$. The black color of the surface limited unwanted light reflection from the wall. At the flat plate leading edge the laminar-turbulent transition was imposed by sandpaper.

Measurements were taken, with a PIV system, which consisted of a 1280x1024 pixels high speed NanoSense MKIII CMOS camera and a continuous Spectra-Physics Argon-Ion laser, with a maximum emitted power of 6 W. The maximum acquisition rate of the camera, at full resolution, was 1024 Hz. The laser beam was expanded by a cylindrical lens and focused by a spherical lens, forming a light sheet with a thickness of about 0.5 mm. The water was seeded with spherical silicon carbide particles, 2 $\mu$m nominal diameter, which were small enough to follow the flow. PIV measurements were taken in the streamwise wall-normal plane $(x,y)$ intercepting the apex of a row of pyramids and the diagonal of the square base. The physical size of the PIV images was 49.4x39.5 mm$^2$ (1047x840 viscous units) for the smooth wall and 42x33.6 mm$^2$ (932x768 viscous units) for the rough wall. In all the cases, the PIV images covered the whole boundary layer thickness. The PIV analysis was done with the LaVision DAVIS 7.2 software to perform the correlations to obtain the velocity fields. The local particle displacements were determined using an adaptive cross-correlation algorithm. The final interrogation window size was 32x32 pixels, overlap of 50%. The obtained vectors were validated using a minimum peak height in the correlation, which was 1.2 times the height of the second highest peak. Erroneous vectors were substituted applying a moving average filter with a kernel of 5x5 vectors. Each velocity vector is representative of the mean velocity in an area of 0.62x0.62 mm$^2$ (13x13 wall units) for smooth wall and 0.52x0.52 mm$^2$ (12x12 wall units) for rough wall. According to the Ligrani and Bradshaw (1987) criterium, a linear dimension of the measurement probe less than 20 wall units is sufficient to resolve the near wall turbulence.

The camera acquisition rate was 800 frames per second, but for the statistical analysis only one image pair each 100 frames were recorded. Therefore the effective acquisition rate of PIV image pairs was 8 Hz. 3300 statistically independent image pairs were recorded to ensure the convergence of the computed averaged quantities.

The error in measuring the velocity with PIV depends mainly on the quality of the images, the particle image size, and the displacement of the particles from the first to the second image. Since the particle image size was 2–3 pixels and a good contrast between the particle light intensity and the background light intensity was obtained, the error in locating the correlation peak was estimated to be lower than 0.1 pixels. Taking into account the displacement of particles from the first to the second image
of about 16 pixels in the free-stream and 11 pixels in the log region for
the smooth wall (14 pixels and 7 pixels respectively for the rough wall), the
error in measuring the instant velocity is less than 1%. The error becomes
1.41 times higher when calculating the variance of the velocities, whereas it
is negligibly small when calculating mean values.

![Image](image_url)

Fig.1. a) Experimental setup. b) Roughness geometry. Dimensions in millimeters.

### 3. Results and comments

The experimental test conditions are given in Table 1. The values of the
Reynolds number, \( Re_\theta \), based on the boundary layer momentum thickness \( \theta \)
and on the external velocity \( U_e \), are 1471 for the smooth wall and 1234 for
the rough surface. These values characterize the present results in the range
of relatively low Reynolds numbers. As it has been mentioned in the
introduction, one of the goals of the present measurements was to stress, for
the case of a 3D rough wall, the results already available in the literature
towards the range of moderately low Reynolds numbers, where, according to
Townsend’s hypothesis, the wall similarity is not expected to apply. The
external flow velocities are \( U_e = 0.485 \text{m/s} \) and \( U_e = 0.36 \text{m/s} \) respectively for
the smooth and the rough surfaces. It was selected a higher flow velocity for
the smooth wall case in order to compare the two flows at similar values of
\( Re_\theta \). The values of the form parameter, \( H \), assures that the two flows are in
conditions of fully developed turbulence. \( \delta \) is the boundary layer thickness
evaluated at \( U = 0.99 U_e \), where \( U \) is the boundary layer mean velocity. \( \delta \) was
determined taking into account the virtual origin. \( \delta^* \) is the boundary layer
displacement thickness. The ratio between the boundary layer thickness and
the roughness height, \( \delta k = 17.2 \), is much lower than the value \( \delta k > 50 \)
predicted by Jimenez (2004) for outer-layer wall similarity to be expected to
hold, but it is in the limit, \( 16 < \delta k < 110 \), indicated by Flack et al. (2007), in
which the outer-layer flow is not influenced by the wall roughness. The ratio
between the friction velocity and the external flow velocity, \( \frac{U_f}{U_e} \), is about 60% higher for the rough wall case. The friction velocity, \( U_f \), for the rough wall has been obtained by the modified Clauser chart method (Perry and Li, 1990) applied in the range of the velocity profiles from \( y^+ \approx 60 \) to \( y/\delta \approx 0.23 \). The friction velocity for the smooth wall has been evaluated by the Clauser chart, applied in the range from \( y^+ \approx 40 \) to \( y/\delta \approx 0.23 \). The uncertainty in the evaluation of the friction velocity for the smooth wall and the rough wall is equal to 3% and 5% respectively. The coordinate \( y \) is the wall-normal distance from the virtual origin. The apex plus indicates, here and in the following, that the quantity has been normalized with respect to the wall units. The values of the viscous length, \( y^* = v/\nu_f \), where \( v \) is the kinematic viscosity, are respectively \( 4.7 \times 10^{-5} \text{m} \) and \( 4.3 \times 10^{-5} \text{m} \), for the smooth and the rough wall.

<table>
<thead>
<tr>
<th>Wall Surface</th>
<th>( U_e )</th>
<th>( \delta )</th>
<th>( \delta^+ )</th>
<th>( \delta^* )</th>
<th>( \theta )</th>
<th>( H )</th>
<th>( Re_{\theta} )</th>
<th>( U_f )</th>
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<td>Smooth Wall</td>
<td>0.485</td>
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<td>590</td>
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<td>2.99</td>
<td>1.27</td>
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<td>0.36</td>
<td>29.29</td>
<td>676</td>
<td>5.62</td>
<td>3.79</td>
<td>1.48</td>
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<table>
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<th>( C_f )</th>
<th>( k )</th>
<th>( k^+ )</th>
<th>( k^* )</th>
<th>( \delta/k )</th>
<th>( \delta/k^* )</th>
<th>( \varepsilon )</th>
<th>( \Delta U^+ )</th>
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<td>0.0039</td>
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<tr>
<td>0.071</td>
<td>0.01</td>
<td>1.7</td>
<td>8.01</td>
<td>39</td>
<td>185</td>
<td>17.2</td>
<td>3.66</td>
<td>1.27</td>
</tr>
</tbody>
</table>

Tab. 1.

Clauser (1954) and Hama (1954) found that the effect of roughness on the mean flow was confined to the inner layer, causing only a downward shift in the smooth wall log-law, \( \Delta U^+ \), whose value in the present measurements is reported in Table 1. They extended the log-law to the case of rough wall boundary layers as

\[
U^+ = \frac{1}{k} \ln \left( \frac{(y_f + \varepsilon)^+}{y^+} \right) + B - \Delta U^+
\]  

(1)
where k=0.421 is the Karman constant, B=5.2 is the smooth wall log-law intercept, \( y_T \) is the distance from the roughness crest, \( e \) is the distance from the roughness crest to the virtual origin and \( \Delta U^+ \) is the roughness function. The virtual origin is evaluated according to Perry and Li (1990), by essentially performing a least square fit of equation (1) to the measured velocity profile changing the values for \( U_t \), \( e \) and \( \Delta U^+ \).

Nikuradse (1933) defined the equivalent sand grain roughness height, \( k_s \), as the uniform sand roughness height that produces the same roughness function in the fully rough regime. The dependence of \( k_s \) from \( \Delta U \) may be expressed as (see Raupach et al., 1991)

\[
\Delta U^+ = \frac{1}{k} \ln(k_s^+) - 3.5
\]

(2)

Finally \( \delta^+ \) is the Karman number. Ideally the Karman number should be matched to investigate outer-layer similarity between smooth and rough surfaces; otherwise it would be difficult to discern if differences in velocity and turbulence profiles are due to roughness or to Reynolds number effects (Flack et al., 2007). Looking at Table 1, the Karman number for the rough wall is not, unlikely, equal to the one for the smooth wall, but they are at least of the same order of magnitude.

A. Mean flow and turbulence

The following results, from Fig. 2 to Fig. 7, have been obtained averaging the flow values along the \( x \)-direction. Mean velocity profiles plotted in inner variables are shown in Fig. 2 for the two flows. The first measurement point reported in Fig. 2 (and in all the diagrams of this section A) for the rough surface is located just above the pyramid apex, corresponding to \( y^+ = 42 \) (\( y_T/k \approx 0.3 \), \( y/k \approx 1.07 \), \( y/\delta \approx 0.06 \)). Details of the flow closer to the wall will be shown later. The smooth wall Laser Doppler Velocimetry measurements of DeGraaff and Eaton (2000) at \( Re_\theta = 1430 \) are also reported for comparison. Data in Fig. 2 show satisfactory agreement between the present velocity profiles for the smooth wall and the reference data. The rough surface displays a linear log region shifted below the smooth profile of a quantity corresponding to the roughness function, \( \Delta U^+ \). Despite of the relatively low Reynolds numbers, the log regions are clearly identifiable. The value of \( \Delta U^+ \) is reported in Table 1. It is not straightforward to compare this value \( \Delta U^+ = 8.9 \) with the values found by Schultz and Flack (2009) for their wall roughness represented by close-packed pyramids with height
going from 0.30mm to 0.60mm and three different slopes 11°, 22° and 45°. The present experiment refers to the case of much higher values of pyramid height, with a slope of 26°, and much lower values of $Re_\theta$. The Schultz and Flack (2009) values of $\delta k$ range from about 50 to about 100; their $Re_\theta$ range from about 3500 to about 30000. In the present experiment $\delta k=17$ and $Re_\theta=1234$. These values represent the frontier of the geometries and flow conditions of most of the studies published on rough wall turbulent boundary layer. In addition, while in Schultz and Flack (2009) the pyramids are close-packed, in the present geometry there is a gap of $g=1.4$mm ($g/k=0.82$) between the pyramid basis. Due to the extreme values of $\delta k$ and $Re_\theta$ of the present experiment the question arises if the results have to be considered significative for the case of a flow in a fully rough regime or the results have to be classified in the transitional regime. The high value of $\Delta U^+=8.9$ associated to a relatively small value of $k^+=39$ are not in line with the data for uniform sand of Nikuradse (1933) and the data reported in Fig. 9 of Schultz and Flack (2009). According to these data a value of about $\Delta U^+=6$ is associable with $k^+=39$. However, $\Delta U^+$ was found to depend on the height $k^+$ of the elements and on other geometric features, such as their density (Colebrook & White, 1937) and shape. Napoli et al. (2008) associated the value of the roughness function $\Delta U^+$ for a two-dimensional irregular roughness to a new parameter termed the effective slope $(ES)$ defined as follows:

$$ES = \frac{1}{L} \int_L \left| \frac{\partial r}{\partial x} \right| dx,$$

where $L$ is the sampling length and $r$ the roughness amplitude. They concluded that $\Delta U^+$ is strongly dependent on $ES$. Later, Schultz and Flack (2009) observed the same trends of $\Delta U^+$ in function of $ES$ also for their 3D pyramidal roughness. They proposed in their analysis that a regime of transition occurs at $ES \approx 0.35$. For $ES>0.35$ a regime of fully rough flow holds. This corresponds to flow conditions around the roughness element in which the form drag is much higher than the frictional drag. For $ES<0.35$ a waviness flow regime in which the frictional drag is not negligible holds. According to this proposed classification, the present data, corresponding to about $ES \approx 0.4$, are to be considered in the very low limit of the fully rough flow regime. It should be noted that the value of $\Delta U^+=8.9$ for $ES \approx 0.4$ is in a quite good agreement with Schultz and Flack (2009) data.
Fig. 3. Mean velocity profiles in velocity defect form.

Fig. 3 presents the mean velocity profiles in velocity defect form, using classic inner-outer scaling by $U$ and $\delta$, respectively for the velocity defect and for the distance from the virtual origin. Also in this figure, as in the
following figures from 4 to 7, the corresponding smooth wall reference data of DeGraaff and Eaton are reported. The present data for the smooth wall in Fig. 3 are in good agreement with the DeGraaff and Eaton data. The two velocity profiles for the smooth and rough walls satisfactory collapse, starting from about $y/\delta = 0.06$. This suggests that, even in part of the roughness sub-layer region, whose height is evaluated for the present roughness geometry as $y/\delta \approx 5k/\delta \approx 0.2$, the mean flow for the present wall geometry is largely insensitive to the surface conditions.

Fig. 4. Streamwise Reynolds normal stress profiles.

The streamwise Reynolds normal stresses $\langle u'u' \rangle^+$ are plotted against $y/\delta$ in Fig. 4. Uncertainty bars of 4.5% and 7% have been indicated for the smooth wall and the rough wall case respectively. A good agreement is observed between the present data for the smooth wall and the DeGraaff and Eaton data. The comparison between the smooth and the rough wall profiles shows that for this Reynolds stress component the flow appears to be quite insensitive to the surface conditions in the outer-layer above $y/\delta \approx 0.2$. Moreover, observing the results in Fig. 4, the streamwise Reynolds normal stress $\langle u'u' \rangle^+$, for the rough wall, displays significantly lower values near the wall (with respect to the smooth wall case) in the region around $y/\delta \approx 0.1$. A recovery of the streamwise Reynolds normal stress is suggested by the first two measurement points nearer to the wall, with the first point (just above the apex of the pyramids) assuming a value of about $\langle u'u' \rangle^+ \approx 3.5$. 
Moreover, a plateau region in $<u'u'>^+$ is present around $y/\delta \approx 0.1$. The presence of this plateau and the reduction in the peak can be seen as an indicator of a boundary layer approaching the fully rough regime (Ligrani & Moffat, 1986). Previous reported results from other authors have found, for a rough wall, the absence of the peak characterizing the smooth wall $<u'u'>^+$ profile in the buffer layer, at about $y^+ \approx 15$ (see the DeGraaff and Eaton results in Fig. 4). This behaviour has been found in most of the measurements reported in literature and it has been interpreted by Flores and Jiménez (2006) as the consequence of the fact that the wall roughness reduces the coherence of the quasi-longitudinal vortices populating the buffer layer and of the induced low-speed streaks. Moreover, the absence of the $<u'u'>^+$ peak observed by other authors in correspondence to the region where the maximum production of turbulent energy is expected in a boundary layer over a smooth wall was interpreted as an indication that the mechanism of turbulence reproduction in the buffer layer by instability of the low speed streaks (Schoppa and Hussain, 2002) is in part or completely prevented, because of the action of the wall roughness.

Fig. 5 displays the distributions of the wall normal Reynolds normal stresses, $<v'v'>^+$, as a function of $y/\delta$. Although the comparable Reynolds numbers, the present smooth wall results show a small discrepancy with DeGraaff and Eaton data, mainly in the region around $y/\delta = 0.2$, where
DeGraaff and Eaton data do not appear to have smooth behaviour. For all measurement points, the collapse between the smooth wall and the roughened wall appears, also for this stress component, completely satisfactory, with the small observable differences within the evaluated uncertainty. Conversely, for the case of two-dimensional roughened surfaces, it should be noticed that other authors (see e.g. Antonia and Krogstad, 2001, Keirsbulck et al., 2002, Vesely et al., 2009) have observed that the $y$-component of the velocity is more sensitive to the roughness effects than the $x$-component.

The present smooth wall results for the Reynolds shear stress (Fig. 6) are situated in between the DeGraaff and Eaton results and the Spalart (1988) direct numerical simulation results. A relatively good collapse between smooth and rough wall Reynolds shear stresses is observable in the limit of the measurement uncertainty. The largest differences of the order of 7% between the two flows are present in the region around $y/\delta = 0.3$.

In Fig. 7 the production of the turbulent kinetic energy, $-\langle u'v' \rangle^+ (\partial U^+ / \partial y^+)$, is reported in function of the distance from the wall. The agreement between the smooth and the rough wall results is satisfactory starting from about $y/\delta = 0.1$. It should be noticed that in the rough wall case the presence of additional production terms may be not negligible.

![Fig. 6. Reynolds shear stress profiles.](image-url)
The last result and the ones previously shown in Figs. 3 to 6, demonstrate that the wall similarity concept between smooth and rough walls may be expected to confidently hold for a typical three-dimensional wall roughness, also in the case of a relatively low Reynolds number. As previously commented, the extension of the wall similarity hypothesis to the case of large wall roughness, as large as $\delta/k = 16$, was demonstrated by Flack et al. (2007), for $Re_\theta$ ranging from $7.3 \cdot 10^3$ to $13 \cdot 10^3$. Nevertheless, it was not expected by the present authors the quasi complete collapse between mean velocity profiles in velocity defect form and normalized Reynolds stresses over smooth and rough walls, in condition of Reynolds numbers not high enough to imply significant scale separation between the large scale motions and the smaller scale turbulent eddies. Similar conclusion has been drawn by some of the present authors (Vesely et al., 2009) testing in the same water tunnel at the Politecnico di Torino a two-dimensional roughness model ($\delta/k=28.6$), at $Re_\theta=1360$.

**B. Organized motions**

In Fig. 8 an example of PIV results in the streamwise wall normal plane is shown, for the case of the rough wall. This instant flow image shows
features similar to the ones expected in a smooth wall turbulent boundary layer, as described by Adrian et al. (2000), Ganapathisubramani et al. (2003), Onorato et al. (2006). The mean flow direction is from the left to the right. The colour map represents the instantaneous component of the streamwise velocity, $u$. Large zones of the flow field having relatively retarded uniform values of the streamwise momentum, separated by thin regions of large $\partial u/\partial y$, are evident. Trains of clockwise swirling motions representing the signature in the measurement plane of packets of vortical structures (hairpin vortices and skewed quasi-longitudinal vortices) are aligned along the boundaries separating the regions of uniform momentum fluid. As Meinhart and Adrian (1995) suggested, the long region of uniformly retarded flow in each zone is the back flow induced by a packet of hairpins that are aligned in a coherent pattern in the streamwise direction and convected at the same velocity.

![Fig. 8. Instantaneous component of the streamwise velocity. Closed lines highlighted by a circle: swirling motions. Rough wall.](image-url)
The swirling motions, caused by the vortical structures in the flow field, indicated in Fig. 8, have been identified by extracting iso-regions of swirling strength $\lambda_{ci}$ (Zhou et al., 1999), which is defined as the magnitude of the imaginary part of the eigenvalue of the local velocity gradient tensor. The vortex detection via swirling strength is Galileian invariant and does not identify regions of intense shear in absence of rotation. In particular, in order to attribute a rotational sign to the vortical structures, the modified swirling strength parameter $\Lambda_{ci} = \lambda_{ci} (\omega_z/|\omega_z|)$ has been adopted, where $\omega_z$ is the spanwise component of the vorticity vector. Furthermore, following Nagaosa and Handler (2003) and Wu and Christensen (2006), a threshold of $|\Lambda_{ci}| \geq C (\Lambda_{ci})_{rms}$, with $C = 1.5$, has been introduced to limit the influence of the experimental noise associate with the calculation of the velocity gradients from PIV experiments. The calculation of $\lambda_{ci}$ and $\omega_z$ involved the computation of velocity derivatives, which were obtained first applying a polynomial filter of second order with a kernel of 5x5 grid points to the velocity data and, successively, applying a second order central difference scheme.

The described flow features in Fig. 8 are commonly observed in the majority of the PIV images for the case of the rough wall as well as for the case of the smooth wall. No obvious imprint of the wall roughness on the flow in the outer region is evident observing single instant images like the one in Fig. 8. Statistical analysis may enlighten possibly existing quantitative differences between the two flow fields over smooth and rough walls.

In Figs. 9 and 10 contours of constant values of the two-point streamwise and wall normal velocity autocorrelation functions, $\rho_{u'u'}$ and $\rho_{v'v'}$, at the reference distance from the wall $(y/\delta)_{ref} = 0.5$, are respectively displayed, for the cases of the smooth and the rough walls. The shape of the contours $\rho_{u'u'}$ for the rough wall (Fig. 9b) is similar to the one for the case of the smooth wall (Fig. 9a) and to the one found (among others) by Liu et al. (2001) in a channel flow. They appear elongated in the mean flow direction and inclined at a shallow angle to the wall.

A different behaviour of the wall-normal velocity correlation function, $\rho_{v'v'}$ is observed in Fig. 10, for the smooth and the rough walls. The $v'$-correlation appears to have roughly circular contours, in evident contrast to the elongated shapes of the $u'$-component correlation. $\rho_{v'v'}$ contours are compact in both streamwise and wall-normal directions and show an extent relatively small with respect to the $u'$-correlation in the mean flow direction.
Fig. 9. Contours of constant values of the two-point streamwise velocity autocorrelation function. \( \frac{y}{\delta_{\text{ref}}} = 0.5 \). (a) Smooth wall. (b) Rough wall.

Fig. 10. Contours of constant values of the two-point wall normal velocity autocorrelation function. \( \frac{y}{\delta_{\text{ref}}} = 0.5 \). (a) Smooth wall. (b) Rough wall.

The characteristic shape of the streamwise and wall-normal velocity correlations in Figs. 9 and 10, reflects the organization of the near wall eddies into packets travelling coherently with uniform velocity. This is consistent with the conjecture that the flow is dominated by a series of hairpin vortices aligned in the streamwise direction, whose heads lie along a line inclined away from the wall, as claimed by Christensen and Adrian (2001) for a smooth wall. As shown in the instant flow field in Fig. 8, these
structures induce a region of low streamwise momentum extended as long as the packet. Within an individual eddy the streamwise and wall-normal scales are expected to be similar, but the assembly of eddies in a coherent packet has much longer scale. The long extension of the \( u' \)-correlation is associated with the coherent induction of many vortical structures in the packet, while the short \( v' \)-correlation is associated with the scale of individual eddies (Liu et al., 2001). The inclination angle, \( \alpha \), of the contours \( \rho_{u'u'} \) defines the orientation of these dominant flow structures. Looking at the results like the ones in Fig. 9, for \( 0.1 < \left( \frac{y}{\delta_{ref}} \right) < 0.5 \), an inclination angle \( \alpha = 11^\circ \pm 2^\circ \) of the contours \( \rho_{u'u'} \) is found, for both smooth and roughened surface. Very close values of \( \alpha \) have been measured in most published investigations. It should be noted that the inclination \( \alpha \) of the contours \( \rho_{u'u'} = \text{const.} \) depends on the distance from the wall of the correlation reference point and also it assumes slightly different values if it is measured looking upstream or downstream the reference point. Liu et al. (2001) found for smooth wall, in a channel flow, an inclination angle \( \alpha \) of 6-8 degrees. Christensen and Adrian (2001) reported values of \( \alpha \) of 12° and 13° in a turbulent boundary layer over a smooth wall. Nakagawa and Hanratty (2001), analyzing PIV results in a channel flow over a two-dimensional sinusoidal roughness \( \delta k_s \approx 60 \), producing a roughness function \( \Delta U^+ = 8.12 \), measured a slope angle of 9°. Christensen and Wu (2005) reported a value of \( \alpha = 11^\circ \) over a smooth wall in a channel flow. Recently Volino et al. (2007) studied experimentally the outer region structure of turbulent boundary layers on smooth and rough walls founding qualitatively similar turbulent structures in the rough and smooth cases. In particular two-point correlations of the velocity and swirl strength were quantitatively similar for the two flows. Vesely et al. (2009) found a value of \( \alpha = 8^\circ \) both on a smooth wall and on a two-dimensional roughened surface (\( \delta k = 28.6 \)). Conversely, Wu and Christensen (2006), testing surfaces containing a broad range of topological scales produced by deposition of foreign materials on the surface (\( \delta k = 25 - 28 \) and \( \delta k > 40 \)), measured two-point spatial velocity correlation coefficients obtaining results sensitive to the surface topology, as smooth and rough data showed measurable differences.

After founding out that the PIV instant images and the two-point correlations reveal that the outer turbulent structure of the flow over the rough surface is qualitatively similar to what is found for the flow over the smooth wall, in order to examine the influence of the wall roughness geometries on orientation and scales of such large scale structures, direct comparisons between \( \rho_{u'u'} \) correlations for the two flows are shown in
Fig. 11. The extent of the regions where $\rho_{u'u'}$ is 0.6 has been chosen to be representative of the topology and the dimension of the structures. Interesting features are shown both in the roughness sub-layer ($(y/\delta)_\text{ref} \leq 0.2$) and in the outer-layer. Far from the wall ($(y/\delta)_\text{ref} = 0.3$ and 0.5) the contours $\rho_{u'u'} = 0.6$ for the smooth and the rough walls are very close, suggesting similar longitudinal and wall normal scales, when lengths are normalized with respect to the boundary layer thickness.

The streamwise extent of the $\rho_{u'u'} = 0.6$ contours at $(y/\delta)_\text{ref} = 0.3$, for both the smooth and the rough wall cases, defined as in Volino et al. (2007) as twice the distance from the self-correlation peak to the most downstream
location on the contour, is $L_{xx'u'} = 0.43$ for the smooth wall and $L_{xx'u'} = 0.47$ for the rough wall. The same value is reported in Volino et al. (2007) for the smooth wall. The value of the ratio between $L_{xx'u'}$ and $L_{yu'u'}$ (the wall-normal extent of $\rho_{u'u'} = 0.6$) at $(y/\delta)_{ref} = 0.3$ is $L_{xx'u'}/L_{yu'u'} = 2.2$. Volino et al. (2007) measured $L_{xx'u'}/L_{yu'u'} = 2.5$. Conversely, the effect of the wall roughness is evident very near the wall at $(y/\delta)_{ref} = 0.1$ and $(y/\delta)_{ref} = 0.2$, where the longitudinal length scale for the flow over the smooth surface appears larger than the one over the roughened surface.

![Contour plots](image)

Fig. 12. Contours of constant values of the two-point wall-normal velocity autocorrelation function at different values of $(y/\delta)_{ref}$. $\rho_{u'u'} = 0.6$.

This confirms the observation of Bakken et al. (2005). They found that the turbulence integral length scale in the streamwise direction was shorter for the case of rough surfaces with respect to the case of a smooth wall,
indicating a break-up of the near-wall streamwise vortices over the rough wall. Moreover, the present results are also consistent with the Flack et al. (2005) founding. They observing results from quadrant analysis of a flow over two different three-dimensional rough surfaces, sandpaper and woven mesh, show that roughness induced changes to the coherent turbulent structures are confined to a near-wall roughness layer, extending $3k_s-5k$ from the wall. In their paper the ratio $\delta/k$ ranges from 45 for the sandpaper ($\delta/k_s = 62$) to 100 for the mesh ($\delta/k_s = 45$).

The $\rho_{\nu'\nu'} = 0.6$ contours in Fig. 12, associated with the scale of individual eddies, show slightly smaller scales for the rough surface flow with respect to the smooth surface case at $(y/\delta)_{ref} = 0.1$, where the contour for the rough wall assumes a near circular shape, while the contour for the smooth wall appears to be slightly enlarged in the streamwise direction.

This fact can be speculatively related to the ability of the wall roughness to reduce the level of anisotropy, as shown in Smalley et al. (2002). At all the farer distances from the wall the autocorrelation functions for the smooth and the rough walls show comparable values, about indistinguishable at $(y/\delta)_{ref} = 0.3$. The ratio $L_{xy'/y'}/L_{x'y'}$ observable at $(y/\delta)_{ref} = 0.3$ for the smooth and the rough walls (0.93 and 0.8 respectively) in the present research and the ones reported in Volino et al. (2007) and in Vesely et al. (2009) for the smooth wall ($L_{xy'/y'}/L_{x'y'} = 0.91$ for both experiments) are very close.

### C. Roughness sub-layer

As previously observed, results from Fig. 2 to Fig. 7 have been obtained averaging values along the streamwise $x$-direction. It has to be expected that, due the morphology of the wall, the flow does not possess homogeneous properties at least near the rough surface. This will be highlighted in the following results, where flow quantities are locally visualized also in the roughness region, very near the wall surface. It should be pointed out that quantities showed in the following maps, measured in the lower region between two successive pyramids, are affected by an uncertainty larger than the one previously evaluated at the more external region (above the pyramid apex).

In Figs. 13 and 14 the streamwise and the wall-normal components of the mean velocity $\tilde{U}$ and $\tilde{V}$ are visualized in a region covering a wavelength $\lambda_x$.
from the top of one generic pyramid to the top of the next one. The symbol tilde (∼) denotes quantities obtained spatially averaging values in corresponding points of each region covering a wavelength $\lambda_x$. To better clarify the symbolism, observe, as an example, that the quantity $U$ reported in Fig. 2 is obtained averaging $\langle \tilde{U} \rangle$ along the streamwise direction. In both figures 13 and 14, the wall roughness is sketched. The white triangles represent the pyramids highlighted by the laser sheet. The dark triangle represents the row of pyramids standing between the laser sheet and the observation video camera. The flow appears to be homogeneous in the streamwise $x$-direction for $y_T > k$ ($y/\delta > 0.08$), where the streamwise component shows an almost constant value along the $x$-direction, function of the distance from the wall, while the component in the $y$-direction assumes vanishing values. In the pyramid region the flow is dominated by the geometry of the roughness. Even if the flow field is not completely observable because in part shaded by the row of the dark pyramids sketched in the figures, regions of low values of $\langle \tilde{U} \rangle$ are evident downstream the pyramid in the region closer to the their base. This flow clearly contributes to the increased drag of the roughened surface with respect to the smooth one. A positive high value of the $\langle \tilde{V} \rangle$ component is observed in Fig. 14a near the pyramid upstream side (the ascendant part of the pyramid), followed by a negative value in the downstream side (the descendant part of the pyramid). To see some flow detail just above the pyramid apex, in Figs. 13b and 14b the streamwise and wall normal components of the mean velocity are displayed in function of $x$, at $y_T/k=0.33$. A peak of $\langle \tilde{U} \rangle$ is observed at $x/\lambda_x \approx 0.1$, followed by a decrease in the region between the two pyramids. A velocity minimum (about 7% lower of the peak) is observed at $x/\lambda_x \approx 0.8$ just upstream the pyramid. Similar low velocity in the upstream side of the pyramid is observed in the Hong et al. (2011) measurements, at about the same distance from the top of the roughness. The $\langle \tilde{V} \rangle$ velocity at $y_T/k=0.33$ shows a much higher variation along the wavelength (Fig. 14b), displaying a negative value of $-0.05$ at $x/\lambda_x \approx 0.35$, just downstream the pyramid, near its foot, and a positive peak of $\langle \tilde{V} \rangle \approx 0.53$ near the top of the ascendant side of the pyramid.
Fig. 13. Streamwise component of the mean velocity. a) Planar distribution. b) Distribution in function of $x$, at $y_{r}/k=0.33$.

The streamwise component of the Reynolds normal stresses is visualized in Fig. 15. The range of the color bar in Fig. 15a, and in the following figures, has been set in order to highlight peak values. $<u'\overline{u}'>$ shows about
homogeneous behavior in the x-direction far from the wall. In contrast, as expected, is dominated by the wall geometry in the region of the pyramids. It

Fig. 14. Wall-normal component of the mean velocity. a) Planar distribution. b) Distribution in function of $x$, at $y_T/k=0.33$. 
assumes downstream the pyramid a peak value very close to the one measured in the case of smooth wall, in the buffer layer, at about the same Reynolds number, $<u'u'^+> \approx 7.7$ (see Fig. 4). Much more lower
values are measured along the upstream side, where the flow is accelerated. A variation along the $x$-direction of about 25% is observed in Fig. 15b, at $y_T/k=0.33$. A maximum value of about 3.5 is now observed between two pyramids, in a region much more downstream of the region were a peak is detected below the pyramid apex (Fig. 15a). A minimum peak value of about 2.6 is observable in the ascendant part of the pyramid (Fig. 15b).

![Wall-normal component of the Reynolds normal stresses: a) Planar distribution. b) Distribution in function of $x$, at $y_T/k=0.33$.](image)

Fig. 16. Wall-normal component of the Reynolds normal stresses: a) Planar distribution. b) Distribution in function of $x$, at $y_T/k=0.33$. 
Fig. 17. Reynolds shear stress: a) Planar distribution. b) Distribution in function of $x$, at $y_p/k=0.33$.

In Fig. 16 the wall-normal component of the Reynolds normal stresses is shown. A peak value of about $-\langle \widetilde{v}' \rangle^+ = 1.2$ is observable downstream the pyramid in the lower part, near the pyramid base. Starting from the top of the pyramid and moving along the downstream direction up to $x/\lambda_x \approx 0.4$, it is observable a region of about uniform value of $-\langle \widetilde{v}' \rangle^+ \approx 0.6$, extending in the more internal region. More downstream higher values of $-\langle \widetilde{v}' \rangle^+$ are present.
with a peak value of about 1 at $x/\lambda_x \approx 0.7$ and $y_T/k \approx 0.2$. The streamwise
distribution of $-\langle v'v' \rangle^+$ at $y_T/k = 0.33$ (Fig. 16b) increases about linearly
between a minimum value of 0.7 at $x/\lambda_x \approx 0.2$ to a peak value of 1.05 at
$x/\lambda_x \approx 0.75$.

Fig. 18. Turbulent kinetic energy. a) Planar distribution. b) Distribution in function
of $x$, at $y_T/k = 0.33$. 
In Fig. 17 the Reynolds shear stress is displayed. \( \langle<u'v'> \rangle^+ \) shows different behaviors in the two sides of the pyramids: higher values in the downstream side, with a peak of \( \langle<u'v'> \rangle^+ \approx 1.6 \) and about half of this value in the upstream side. In the region between two successive pyramids \( \langle<u'v'> \rangle^+ \) also shows a relatively high value around \( \langle<u'v'> \rangle^+ \approx 1 \). Farer from the wall (Fig. 17b), at \( y_T/k=0.33 \), a peak value of \( \langle<u'v'> \rangle^+ \approx 1.07 \) is positioned at about \( x/\lambda_x \approx 0.7 \), much more downstream with respect to the position of the peak detected nearer the wall.

The local values of the turbulent kinetic energy (TKE) in Fig. 18 also show a non homogeneous behavior in the region between the roughness elements. A peak value as high as \( \langle<TKE> \rangle^+ \approx 4 \) is observable in the same region where \( \langle<u'u'> \rangle^+ \) and \( \langle<u'v'> \rangle^+ \) show their peak values. Also in this case at \( y_T/k=0.33 \) (Fig. 18b) the distribution along the streamwise direction shows the highest value much downstream, between the two pyramids, at \( x/\lambda_x \approx 0.55 \).

In Fig. 19 the spanwise component of the mean vorticity is shown in the region containing two successive pyramids. The continuum lines are swirling strength isolines, \( \lambda_{ci} = \text{const.} \), and the vector fields are the mean flow velocity after subtracting a convective velocity \( U_c=33\%U_e \) and \( V_c=1.4\%U_e \). The high value of the mean vorticity near and just downstream
the top of the pyramids, the closed lines $\lambda_{ci} = \text{const.}$ and the two velocity vectorial fields identify regions where flow swirling motions are present in the mean representation of the flow. Far from the pyramids the vorticity appears sufficiently homogeneous in the $x$-direction and function of the distance from the wall. It may be argued that the swirling patterns visualized in the mean representation of the flow, near the top of the pyramids, represent the signature of complex three-dimensional structures, interacting with the wall roughness. Observing the dynamics of the time resolved sequence of PIV flow images, they can be tentatively interpreted as the result, in the averaging process, of two combined effects. The first effect is due to the transit of previously generated vortical structure travelling along the wall (as it can be observed in the sequences of instantaneous velocity fields) and retarded when interacting with the roughness element. The second effect is related to the vortices locally originating from the interaction of the incoming flow with pyramids and successively convected downstream. The last process can be associated to a specific near wall turbulence production mechanism in the roughness layer. This turbulence production mechanism is experimentally documented by Acarlar and Smith (1987) in the case of a single wall roughness element consisting of a hemispherical bump disturbing the flow. They observed the steady formation of a continuous stream of hairpin vortices shed downstream.

A similar mechanism of turbulence structure production was hypotized by Djenidi et al. (2008) also in the case of a two-dimensional roughness.
They observed unstable shear layer vortices shed at the trailing edge of the roughness element and convected downstream before impacting on the following roughness element. More recently, as already indicated in the introduction section, Hong et al. (2012) and Talapatra and Katz (2012) proposed the developing of $U$-shaped quasi-streamwise vortices near the top of the pyramids, as the spanwise vorticity is stretched in regions of high streamwise velocity between the pyramidal roughness elements.

The results in figures 20 and 21 contribute to the understanding of the behaviour of the turbulence in the region near the downstream side of the pyramids, where $\langle u'u' \rangle^+ , \langle TKE \rangle^+ , \text{ and } -\langle u'v' \rangle^-$ show their peak values. The figures represent the conditional averaged Reynolds shear stresses obtained using the quadrant method. The flow field is decomposed into four quadrants based on the orientation of the velocity fluctuation vectors. Namely: Q1 with $u' > 0 , v' > 0$ ; Q2 with $u' < 0 , v' > 0$ (ejection) ; Q3 with $u' < 0 , v' < 0$ ; Q4 with $u' > 0$ and $v' < 0$ (sweep). Results are shown relatively to events in the second and fourth quadrants only, because of their role in the bursting process. As usually is done in the classical quadrant method (Lu and Willmarth, 1973), strong events are detected when

$$|u'v'|_Q \geq H < u'u' >^{1/2} < v'v' >^{1/2}$$

In particular in figures 20 and 21 the Reynolds shear stress have been conditionally averaged on the base of ejection events (Fig. 20) and sweep events (Fig. 21) detected at $x/\lambda_x = 0.2$ and $y_T/k = -0.25$, about in the middle of
the downstream side of the pyramids, where a strong turbulence activity has been measured. In the figures the swirling motions at the top of the pyramids, already commented, are highlighted. The swirling motion appearing about at the top of the sketched dark pyramid, in Fig. 20, suggests that the ejection events are mainly produced by the transit of a vortical structure, that in the conditionally averaged flow field, appears located downstream the pyramid, between \( y_T/k = 0 \) and \( y_T/k = 1 \). Also the sweep events (Fig. 21) appear to be induced by a vortical structure now upstream the conditioning detection point. The representation in Fig. 21 of this structure is not completely evident because of its interaction with the swirling motion present above the top of the pyramid. Actually, raising the conditioning detection point from the wall, the two structures may be seen completely independent.

4. Conclusions

PIV measurements at relatively low Reynolds numbers in a turbulent boundary layer over a 3D-roughened surface, consisting of pyramidal rows, have been presented and compared to those for a smooth wall. Measurements have been taken in a streamwise wall normal plane intercepting the apex of a row of pyramids and the diagonal of the square base. Mean flow and Reynolds stresses measurements, averaged along the streamwise \( x \)-direction, support the Townsend’s wall similarity hypothesis in spite of the relatively high value of the roughness thickness \( (\delta/k = 17.2) \) and the relatively low Reynolds numbers, \( Re_\theta = 1471 \) and 1234, respectively for the smooth and rough wall.

Contours of two-point streamwise velocity autocorrelation function, for \( \rho_{u'u'} = 0.6 \), near the wall at \((y/\delta)_\text{ref} = 0.1\) \((y_T/k=1)\), show for the case of the rough surface a smaller streamwise scale with respect to the smooth wall case. This indicates a loss of longitudinal coherence due to the action of the surface roughness. It may be consequently deduced that the organization of the flow in the buffer layer as longitudinal highly coherent high- and low-speed streaks is strongly influenced by the wall roughness and that the velocity streaks are weakened or even destroyed. This loss of longitudinal coherence associated with the different near wall behaviour of \(<u'u'>\) is an indication that the mechanism of turbulence reproduction in the buffer layer by instability of the low-speed streaks (Schoppa and Hussain, 2002) is in part or completely prevented, because of the action of the surface roughness. The interaction between the incoming flow and the roughness element can
be at the basis of a new possible specific turbulence production mechanism close to the wall. This was argued also by Djenidi et al. (2008) in the case of observed unstable vortical structures originating downstream a two-dimensional roughness. Such possible mechanism is expected to be dependent on the roughness geometry, and its penetration far from the wall will depend on the magnitude of the roughness element with respect to the largest turbulent length scale. The collapse of the two-point streamwise velocity autocorrelation functions between the smooth and the rough wall flows, observed outside the roughness sub-layer, indicates that the flow far from the roughened surface is organized in packets of vortical structures travelling coherently, as in the case of a canonical smooth wall. This is confirmed by the visual analysis of the PIV snapshots (see Fig. 8). The similitude of the turbulence organization between the rough and the smooth wall flows supports the idea that the parent-offspring scenarios (Zhou et al., 1999) for the turbulence regeneration mechanism (involving the reproduction of new vortices by direct action of existing vortices) is likewise active in the case of rough walls as for the case of a smooth wall.

Measurements very near the wall, in the roughness sub-layer, have pointed out the non-homogeneity of the flow in the streamwise direction. The different flow behaviour in the ascendant and descendent part of the pyramids and in the region between two consecutive pyramids has been visualized. Low values of the streamwise component of the mean velocity and high values of the streamwise component of the Reynolds normal stresses, turbulent kinetic energy and Reynolds shear stresses are present in the region near the downstream side of the pyramids. Moreover, regions of swirling motions have been identified, in the mean representation of the flow, near and just downstream the top of the pyramids. It has been argued that these swirling patterns represent the signature of complex three-dimensional structures, interacting with the wall roughness. Observing the dynamics of the time resolved sequence of PIV flow images, they have been tentatively interpreted as the result, in the averaging process, of two combined effects. The first is due to the transit of previously generated vortical structures travelling along the wall and retarded when interacting with the roughness element. The second effect is related to the vortices locally originating from the interaction of the incoming flow with the roughness element and successively convected downstream. The last process can be associated to a specific near wall turbulence production mechanism in the rough layer.

Finally, a classical quadrant method analysis was performed, in order to contribute to the understanding of the behaviour of the turbulence in the region near the downstream side of the pyramids, where \( \langle u' \rangle^+ \) and \( \langle TKE \rangle^+ \)
and $-\langle \tilde{u} v' \rangle$ show their peak values. Reynolds shear stresses, conditionally averaged on the base of ejection and sweep events detected at $x/\lambda_x = 0.2$ and $y_\tau/k = -0.25$, reveals that ejections are mainly produced by the transit of a vortical structure, that in the conditionally averaged flow field, appears located downstream the pyramid, while sweep events are induced by a vortical structure now located upstream the conditioning detection point.

References


